Optimal locations of Power Quality Monitors Considering Voltage Sag Constraints

الأماكن المثلى لمراقبة جودة القدرة باعتبار قيود انخفاض الجهد

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الملخص :

يتناول هذا البحث مشكله تحديد الأماكن المثلى لمراقبة جودة القدرة الكهربية, حيث تعتمد الطريقة المقترحة على البرمجة الخطية لتحديد أقل عدد من المراقبين وأماكنهم عند قيم مختلفة لمستويات الجهد وحالات تشغيل مختلفة لمنظومة القوى الكهربية بطريقة تضمن السيطرة على جميع مواضع الخطأ. وقد أثبتت خصائص الأداء أن الطريقة المقترحة تعتبر إحدى الطرق التنافسية بناء على مقارنتها بطرق أخرى حيث تضمن الحصول على متطلبات الملاحظة الكاملة للنظام, وتم تطبيق الطريقة المقترحة على منظومة قوى كهربية قياسية مكونة من 30 قضيب (IEEE 30-bus test system).

ABSTRACT:

This paper addresses the problem of identifying the optimal locations for power quality monitors (PQMs). A proposed approach is based on integer linear programming (ILP) to solve PQMs problem. It gives the minimum number of PQMs and their locations at variable voltage threshold values. The proposed method solves the PQMs problem for different network configurations that ensures all fault positions are captured. Performance characteristics prove that the proposed method is a competitive one compared to other methods in the literature and guarantee complete observability requirements of the whole power system. The method is efficiently applied to IEEE 30-bus network. The proposed method is implemented in MATLAB environment.

Keywords: power quality monitoring, voltage sag, and Integer optimization.

1. INTRODUCTION

Power quality (PQ) has been treated as a prominent issue which demands utilities to deliver a good quality of electrical power to users especially for industries which have sensitive equipments. Among all the power disturbances, voltage sags are the most frequent and give severe impact on sensitive loads [1].

Voltage sags are the most frequent disturbance which causes severe impact on sensitive loads. According to IEEE standard 1159-1995, voltage sag is defined as a decrease in rms voltage between 90% and 10% of nominal voltage for a time duration between 0.5 cycles and one minute [2].Voltage sags and swells are normally due to the switching of large load (motor starting, transformer energizing, etc.) or due to short-circuits [3]. Power systems have non-zero impedances, so every increase in current causes a corresponding reduction in voltage. Usually, these reductions are small enough that the voltage remains within normal tolerances. But when there is a large increase in current, or when the system impedance is high, the voltage can drop significantly [4].

Installation of metering and monitoring systems has been growing rapidly for several reasons such as the need for automated metering and customer billing [5]. Power quality monitoring (PQM) should be applied to make sure that high quality of electricity is supplied to customers [6].In the traditional PQM practice, monitors are installed at all buses in a power distribution network to monitor voltage sags. But, it is needed to reduce the number of monitors and the total cost of monitoring system [7]. It is also required to reduce redundancy of data being measured by monitors. Thus, it is necessary to determine the best locations of monitors such that any voltage sag is captured.

Recently, many studies have focused on solving the PQMs placement problem [7] - [11].

The PQMs locations must guarantee observability of the entire system and capture any voltage sag event by at least one monitor [12]. Hence, PQMs placement methods can be classified into four main methods, namely, monitor reach area (MRA), covering and packing (C&P), graph theory (GT), and multivariable regression (MVR) [13]. In 2003, a new concept was introduced for the optimal location of PQMs known as MRA [9]. MRA is the area of network where a monitor can detect voltage sags caused by short-circuit faults. To identify the optimal locations of meter, optimization problem is formulated and solved by genetic algorithm (GA), and integer linear programming [10]. In 2009, an approach was addressed for optimal location of voltage sag monitors based on the monitor reach matrix (MRM) by solving analytical expressions. It can give complete observability of the power system for any type of fault (balanced or unbalanced) [11].

A technique based on MRA and the fault location observability analysis (FLOA) is applied for determining the monitor placement sequence and evaluating the effectiveness of suboptimal monitoring programs [14].

In this paper, a direct method for solving the optimal PQMs placement problem in power system is presented. The method is based on applying ILP algorithm which gives the same results and achieves the different objectives such as maximizing the observability and minimizing the number of monitors and installation costs. In the proposed algorithm, the observability concept is introduced which is based on the modified monitor reach area (MMRA). In this study, the voltage threshold value (α) is suggested to be variable (from 0.9 to 0.1) p.u. The proposed algorithm is applied to the IEEE 30-bus test system.

2. PROBLEM FORMULATION

A modified version of the PQMs placement method given in [9] is adopted in this work. This method is based on MRA of potential monitoring locations. All MRAs of a network can be modelled as a binary matrix of N_b rows and F columns.

N_b is the number of buses of the network, and F is the number of fault positions. The residual voltages are saved in matrix called as the Fault Voltage (FV) matrix. Its columns (j) represent the bus numbers of residual voltages and its rows (k) refer to the position of a sag-producing fault of a specific type [15]. Then, the MRA matrix can be obtained by comparing all the FV matrix elements for each phase against a threshold value, α. The corresponding element of the MRA matrix is set as 1, when the p.u voltage goes below or equal to α in any phase. Otherwise, it is set as zero. The MRA matrix could be obtained as:

$$\mathrm{MRA}(j,k)=\begin{cases}1 \ , \ \text{if } \ \mathrm{FV}(j\,,k)\le\alpha \\ 0 \ , \ \text{if } \ \mathrm{FV}(j\,,k)\succ\alpha\end{cases} \quad \forall\, j\,,k \tag{1}$$

In this study, a modified monitor reach area (MMRA) is presented to make it applicable for both distribution and transmission systems. MMRA considers one of the important issues which help to define the minimum numbers and locations of the monitors and make sure that this number is enough for covering the system.

Therefore; in this paper the MMRA is built based on the concept of path graph theory [16]. Similar to MRA and FV matrices, the M matrix column is correlated to bus number and its row is correlated to fault location. The matrix is filled with 1 (one) when there is a path from generator bus to a particular bus in the system and 0 (zero) otherwise. Thus, the MMRA matrix is given by:

$$\mathrm{MMRA}(j,k) = \mathrm{MRA}(j,k) \bullet \mathrm{M}(j,k) \quad \forall j,k \quad (2)$$

$$\mathrm{M}(j,k) = \begin{cases} 1 \text{, if there are apath from generator to bus} \\ 0 \text{, otherwise} \end{cases} \quad (3)$$

Fig. 1 shows an example of a particular row in M matrix for a radial system with two power sources. When a fault happens at bus 3, two generators are connected to the system at bus 1 and bus 5. In this case, there is a path from generator bus (bus 1) to buses 1, 2 and 3 and there is a path from generator bus (bus5) to buses 5, 4 and 3 but not for the buses 6 and 7. Therefore M matrix is [1 1 1 1 1 0 0].

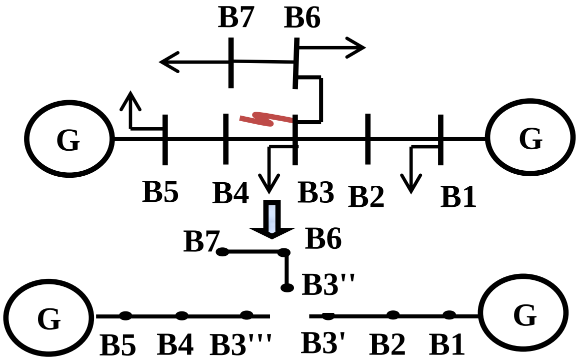

Fig.1: A radial system with two power sources

a) Decision Vector

To represent the binary decision vector, the meter placement vector (X) is formed. This vector indicates positions of monitors in power network. Its elements take only 0 or 1. The value 0 indicates that no monitor is needed at bus n whereas the value 1 indicates that a monitor should be installed at bus n.
Thus, the X vector is described by:

$$\mathrm{X}(n) = \begin{cases} 1 \text{, if PQM is required at bus n} \\ 0 \text{, if PQM is not required at bus n} \end{cases} \quad \forall n \quad (4)$$

b) Objective function

The objective function (O.F) of the optimization is minimizing the number of required monitors and it's described by:

$$\mathrm{O \cdot F} = \mathrm{Min} \sum_{n=1}^{N} \mathrm{X}(n) \quad (5)$$

c) Optimization Constraints

Multiplication of the MMRA matrix by the transposed X matrix gives the number of monitors that can detect voltage sags due to a fault at a specific bus. If one of the resulting matrix elements is 0 then no monitor is capable of detecting sag caused by faults at a particular (the corresponding) bus. Whereas if the value is greater than 1, it means that more than one monitor have observed a fault at the same bus. Since each fault must be observed by at least one monitor, the constraint is given by:

$$\sum_{i=1}^{k} \mathrm{MMRA}(k,j) \bullet \mathrm{X}(i) \geq 1 \quad \forall k,j \quad (6)$$

The solution of the optimization problem described by (5) and (6) provides the minimum number of monitors and their locations required to detect all the voltage sags in the network.
In this paper, after finding the solution of optimization problem, it is found that the number of monitors is very large so a method to minimize this number is used; this method is constructed based on the topology of the system and the data of transmission line connections in the system. For example if a location of monitor in the system at buses 1, 3, 5, 6 and 8 and the topology matrix have a connection between buses 1, 3 and 5, 8. The new location of monitors after applying the topology matrix is at buses 1, 5 and 6.The topology matrix (T) is formed as:

$$\mathrm{T}(j,k) = \begin{cases} 1 \text{, if bus j and bus k are connected} \\ 0 \text{, otherwise} \end{cases} \quad (7)$$

3. Direct Algorithm

This paper presents a direct algorithm to find the minimum number of monitors and their locations. The advantages of this method is considering a direct method, easier than integer linear programming and taking a minimum time for solving the optimization. The proposed algorithm for allocating PQMs is summarized as follows:

Step 1: Evaluate the monitor reach area matrix (MRA) as shown in (1).

Step 2: Sum the columns of MRA matrix to give a vector named column.

Step 3: Find the maximum value in column and its order (bus). Locate the first PQMs at this bus number (B).

Step 4: If MRA (I, B) =1 where I denotes the row number, Multiply the rows elements of the I^{th} row by zero.

Step 5: Repeat step 2 to step 4 until all elements in the matrix MRA equal zero.

Step 6: The number of PQMs and its locations is obtained, but this number is too much.

Step 7: Evaluate the MMRA matrix as shown in (2), (3).

Step 8: To guarantee if this number and locations is enough to cover the whole system, we should apply the observability vector, data redundancy and total cost saving.

Step 9: The minimum number of monitoring is obtained.

The flow chart describing the overall optimization problem of PQMs placement is shown in Fig.2.

4. OPTIMAL PQMS PLACEMENT IN CONTINGENCY CONDITIONS

The algorithm in this section is the same algorithm which derived in normal condition but there are some modifications in the constraint during a single PQM loss or line outage. The objective function can be expressed mathematically as:

$$\text{Min} \sum_{i=1}^{N} X(i) \qquad (8)$$

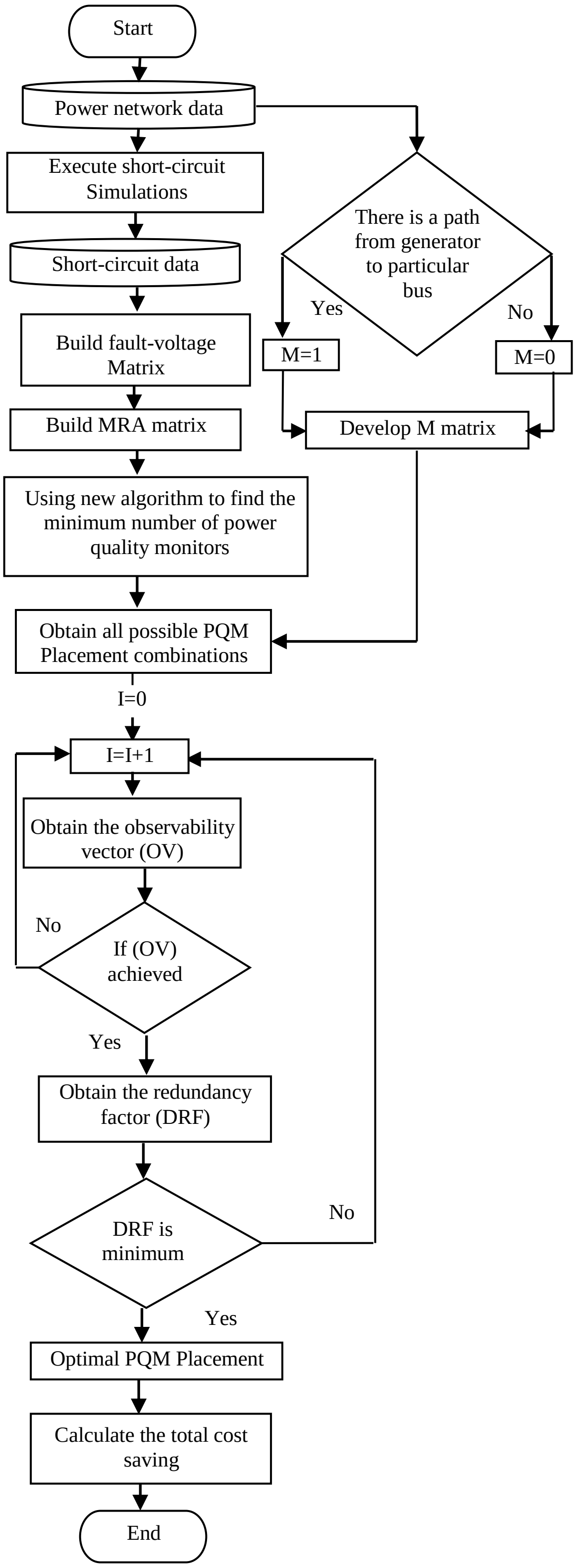

Fig. 2: Overall optimization flowchart

a) Loss of single PQM

The contingency in single PQM effects system observability. In this part the objective is to minimize the total number of PQMs and to save the observability of the system. In order to maintain network observability during a loss of single PQM each bus of the system must be observable from two monitors [17]. The PQMs problem can be formulated as an integer linear program. The objective function can be written as:

$$\text{Min} \sum_{i=1}^{N} X(i) \tag{9}$$

Subject to:

$$\sum_{i=1}^{k} \text{MMRA}(k\ ,j) \bullet X(i) \geq 2 \qquad \forall\, k\,,j \tag{10}$$

b) Loss of single branch

The observability analysis is performed to consider the impact of a branch outage on network observability. In order to maintain network observability during a line outage, each bus of the system must be observable from two paths.

It is clear that if one of the paths is lost (single line outage), that bus is still observable through the other path [18].

5. CASE STUDIES

a) Test System

The IEEE 30-bus test system [19] is used to test the proposed technique for optimal PQMs placement. The IEEE 30- bus test system has 6 generators, 19 fixed loads and 41 branches as shown in Fig.3. This test system has three different voltage levels, that is, buses 11 and 13 at 11 kV, buses 1 to 9 and 28 at 132 kV, and the remaining buses are at 33 kV. The obtained results are compared with those obtained in previous work using integer linear programming.

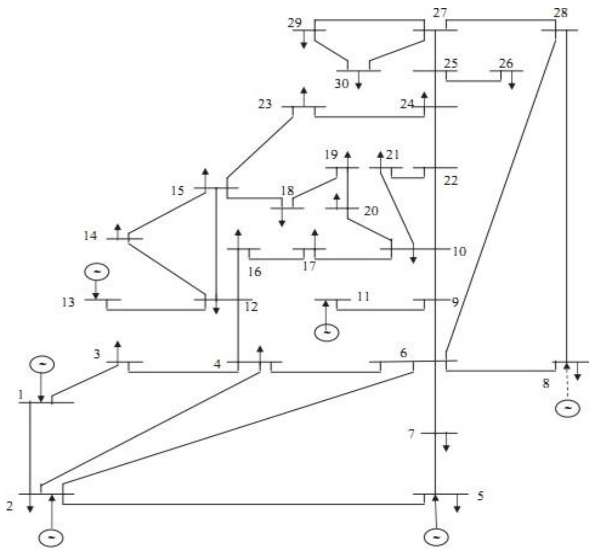

Fig.3: One-line diagram of IEEE 30- bus system

b) Results and Discussion

For PQMs placement, several types of short circuit studies with zero ohm fault resistance are conducted at each bus. This enables to determine the relationship between the unmonitored or estimated bus voltages and the monitored or observed bus voltages [13].

After applying the optimization algorithm using equations from 1 to 6, it is found that only one monitor is enough to observe the whole system when α value is set to 0.9 p.u.

Table I shows the optimal number of PQMs at different α values. These results are obtained from applying the sequence of direct algorithm but not applying topological matrix.

Table II and table III show optimal PQMs placement results of the 30-bus system for different values of α after applying the topological matrix proposed in (7).

To check the results which give the optimal number and location for the PQ monitors we must apply the following factors:-

i. Observability Vector

The observability vector (OV) is defined as a vector referring to how many times each bus in the system has been observed [20].

It checks the capability of a given monitoring system to make the whole system observable, and is given by:

$$O.V = MRA \cdot X \tag{11}$$

Fig. 4 shows the observability vector for test system at different values of α.

TABEL I: The optimal number of PQMs at different α values

α	Number of PQMs	PQMs Placement (bus)
0.9	1	1
0.8	1	4
0.7	2	4 6
0.6	5	2 7 9 13 28
0.5	5	2 5 8 11 12
0.4	6	2 5 6 11 12 13
0.3	8	1 2 4 5 6 11 12 13
0.2	17	1 2 4 5 6 7 8 9 10 11 12 13 16 18 20 23 24
0.1	23	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17 18 19 20 23 24 25 27

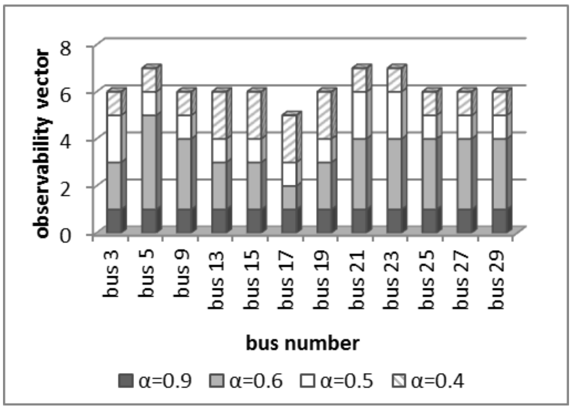

Fig. 4: Observability vector at different values of α

ii. Data redundancy

The data redundancy is one of the major problems in power quality monitors because of the network voltage can be observed by two monitors or more. The data redundancy factor (DRF) is defined as how many times the state variables are measured or calculated and it is given in [6] by:

$$DRF = \frac{\text{sum of number of observing state variables}}{\text{number of state variables}} \tag{12}$$

To avoid counting events more than once, one should minimize the redundancy.

iii. Total cost saving percentage

Percentage total cost saving (TCS) is given in [13] by:

$$TCS\ \% = \left[1 - \frac{N}{M}\right] * 100 \tag{13}$$

Where, N is the number of PQMs installed in the test system and M is the total bus number.
If PQMs have equal cost and monitor threshold α is 0.1p.u, the calculated TCS value is 66.67%.

Table IV shows the optimal number of monitors for different types of faults at different α values, while table V shows the optimal location of monitors for different types of faults at different α values.

In this part a comparison between three cases
a) normal case.
b) Contingency in three-transmission line.
c) Adding three transmission lines to the system.
Table VI shows the optimal number of PQMs at different α values after applying the contingency in lines 1-2, 2-6 and 25-27. Table VII shows the amendment of results in table VI after applying the topology matrix at α equal 0.1 p.u. After applying the third case which three lines 3-13, 6-11and 24-30 are added .It is found that there are 25 PQMs at α= 0.1p.u. This number is higher than the number of monitors in normal case and the results are shown in table VIII. When applying the topology matrix, the number of monitors deceases to 11 monitors and the results are shown in table IX.
Finally, the numbers of monitors that cover the whole system and cover the three cases above are eleven monitors.

TABEL II: PQMs placement after applying the topology matrix at different α

α	Number of PQMs	PQMs Placement (bus)
0.9	1	1
0.8	1	4
0.7	2	4 6
0.6	5	2 7 9 13 28
0.5	4	2 8 11 12
	5	2 5 8 11 12
0.4	3	2 11 12
	4	5 6 11 12
	5	2 5 6 11 12
	6	2 5 6 11 12 13

TABEL III: PQMs placement after applying the topology matrix at α = 0.1

α	Number of PQMs	PQMs Placement (bus)
0.1	10	3 5 6 11 12 17 18 20 23 25
	12	1 2 3 7 8 9 12 17 18 20 23 25
	15	1 2 3 5 6 7 8 9 11 12 17 18 20 23 25
	17	1 2 3 4 5 6 7 8 9 10 11 13 15 16 19 24 27
	23	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17 18 19 20 23 24 25 27

TABEL IV: Optimal number of monitors for different types of faults at different α values

A	SLGF	LLF	DLGF	3PF
0.9	1	1	1	1
0.8	1	2	1	1
0.7	2	3	2	3
0.6	5	6	5	5
0.5	5	17	5	5

TABEL V: Optimal location of monitors for different types of faults at different α values

α	SLGF	LLF	DLGF	3PF
0.9	1	2	1	1
0.8	4	6 12	4	4
0.7	4 6	2 9 12	6 12	6 9 12
0.6	2 7 9 13 28	2 5 6 11 13 15	2 7 9 13 28	2 7 9 13 28
0.5	2 5 8 11 12	1 2 4 5 6 7 9 10 11 12 13 15 18 20 23 24 25	2 5 6 11 13	2 5 6 11 13

c) Comparison to other methods

The direct algorithm was applied to identify the number and location of PQMs. It is compared to other reported methods [13, 15, 21 and 22].

In [13], The MVR, the C&P and the MRA methods are applied to solve the optimal PQMs placement problem. The optimum number of monitors is found to be three, ten and eight PQMs for the MVR, C&P and the MRA methods at α=0.6 p.u respectively.

Table X compares the performance of the direct method to the various methods. At α=0.6, the calculated TCS percentage values are 90%, 73.4% and 66.7% for the MVR, MRA and C&P methods, respectively. These values imply that the MVR, the MRA and the C&P methods can scan the rms voltage magnitude with 3, 8 and 10 PQMs, thus reducing the cost of PQMs by 90, 73.4 and 66.7 percent, respectively.

In [15], a GA is used for solving the optimal PQMs placement problem. In [21], a quantum-inspired binary particle swarm optimization (QBPSO) and adaptive quantum-inspired binary particle swarm optimization (AQBPSO) are applied for solving the optimal PQMs placement problem.

Table XI compares the performance of the direct method to GA, QBPSO and AQBPSO. In this table, it is found that only one monitor is enough to observe the entire system when α value is set to o.85 p.u. The optimal number of monitors when α value is set to 0.55 p.u. is 8 monitors. By comparing this number to proposed method, it is found that the number is very more and not achieve high cost saving and has high redundancy.

A fuzzy genetic algorithm (FGA) was applied in [22], the optimal number of monitors is 7 but this number is not enough for observing the whole system. Though the proposed PQMs placement method is simple, fast and its performance surpasses most of other reported methods.

TABEL VI: Optimal number of PQMs at different α values after applying the contingency in lines 1-2, 2-6 and 25-27

α	Number of PQMs	PQMs Placement (bus)
0.9	1	3
0.7	2	4 6
0.5	6	1 2 5 6 11 13
0.3	9	1 2 4 5 6 11 12 13 20
0.1	20	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17 18 20 23 24

TABEL VII: PQMs placement in case of contingency of three lines after applying the topology matrix at α equals 0.1

α	Number of PQMs	PQMs Placement (bus)
0.1	9	3 5 6 11 12 17 18 20 23
	10	1 4 5 8 9 13 15 16 20 24
	11	1 2 3 7 8 9 12 17 18 20 23

TABEL VIII: Optimal number of PQMs at different α values after adding three lines 3-13, 6-11, 24-30

α	Number of PQMs	PQMs Placement (bus)
0.9	1	1
0.7	1	6
0.5	4	2 4 5 11
0.3	7	1 2 4 5 6 11 13
0.1	25	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17 18 19 20 23 24 25 27 29 30

TABEL IX: PQMs placement in case of adding three lines after applying the topology matrix at α = 0.1

α	Number of PQMs	PQMs Placement (bus)
0.1	11	1 4 5 8 9 13 15 16 19 24 27
	12	2 3 7 8 9 12 17 18 20 23 25 29
	13	1 2 3 7 8 9 12 17 18 20 23 25 29
	15	1 2 3 5 6 7 8 11 12 17 18 20 23 25 29

6. CONCLUSION

This paper presents a direct method based on MRA for finding the optimal number and location of power quality monitors. In the proposed method the fault position has been considered to be the buses for both balanced and unbalanced faults. The proposed technique has been tested on the IEEE 30-bus test system for finding the best optimal PQMs placements at different voltage threshold values from 0.1 to 0.9 p.u. The method ensures complete observability of the network by applying three performance indices for different types of faults. Moreover, the proposed method is competitive with other methods found in the literature and characterized by their simplicity and applicability for wide range of voltage sag levels for different fault types.

TABEL X: Performance comparison to MVR, MRA and C&P methods

α	MRA method [13]				MVR method [13]				C&P method [13]				direct method			
	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF
0.8	3	90	76.67	1.03	3	90	76.67	1.03	10	66.67	93.33	3.8	1	96.67	100	1
0.7	6	80	86.67	1.07	3	90	76.67	1.03	10	66.67	93.33	3.8	3	90	100	2.37
0.6	8	73.33	86.67	1.67	3	90	76.67	1.03	10	66.67	93.33	3.8	5	83.33	100	1.27

TABEL XI: Performance comparison to GA, QBPSO and AQBPSO

A	GA optimization [15]				QBPSO [21]				AQBPSO [21]				direct method			
	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF	No. of PQMs	TCS	OV	DRF
0.85	1	96.67	60	0.6	1	96.67	60	0.6	1	96.67	60	0.6	1	96.67	100	1
0.75	3	90	76.67	1.03	3	90	76.67	1.03	3	90	76.67	1.03	1	96.67	100	1
0.65	6	80	86.67	1.07	6	80	86.67	1.07	6	80	86.67	1.07	3	90	100	2.37
0.55	8	73.33	86.67	1.67	8	73.33	86.67	1.67	8	73.33	86.67	1.67	5	83.33	100	1.27
0.45	11	63.33	90	1.27	11	63.33	90	1.27	11	63.33	90	1.27	5	83.33	100	1.07

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